

## Two-loop Calculations in the MSSM with FeynArts \*

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### Abstract

Recent electroweak two-loop corrections in the Minimal Supersymmetric Standard Model (MSSM) are reviewed. They have been obtained with the help of the programs *FeynArts* and *TwoCalc*, making use of the recently completed MSSM model file for *FeynArts*. Short examples of how to use the two codes together with the analytic result for the  $\mathcal{O}(G_F^2 m_t^4)$  corrections to the  $\rho$ -parameter in the MSSM are presented.

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# Two-loop Calculations in the MSSM with FeynArts

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**Abstract.** Recent electroweak two-loop corrections in the Minimal Supersymmetric Standard Model (MSSM) are reviewed. They have been obtained with the help of the programs *FeynArts* and *TwoCalc*, making use of the recently completed MSSM model file for *FeynArts*. Short examples of how to use the two codes together with the analytic result for the  $O(G_F^2 m_t^4)$  corrections to the  $\rho$ -parameter in the MSSM are presented.

## INTRODUCTION

Theories based on Supersymmetry (SUSY) are widely considered as the theoretically most appealing extension of the Standard Model (SM). The Minimal Supersymmetric Standard Model (MSSM) predicts the existence of scalar partners  $\tilde{f}_L, \tilde{f}_R$  to each SM chiral fermion, and spin-1/2 partners to the gauge bosons and to the scalar Higgs bosons, where two Higgs doublets are present in the MSSM. So far, the direct search of SUSY particles at present colliders has not been successful. One can only set lower bounds of  $O(100 \text{ GeV})$  on their masses, see Ref. (1). An alternative way to probe SUSY is to search for the virtual effects of the additional particles. The experimental precision has to be matched with high precision theoretical predictions for the various precision observables. The most prominent role in this respect is played by the  $\rho$ -parameter, see Ref. (2):

$$\rho = \frac{1}{1 - \Delta\rho}, \quad \Delta\rho = \frac{\Sigma_Z(0)}{M_Z^2} - \frac{\Sigma_W(0)}{M_W^2}. \quad (1)$$

The radiative corrections to the vector boson self-energies at zero momentum transfer,  $\Sigma_{Z,W}(0)$ , constitute the leading, process independent corrections to many electroweak precision observables, such as the  $W$  boson mass,  $M_W$ , where

$$\delta M_W \approx M_W/2 \cdot c_W^2/(c_W^2 - s_W^2) \Delta\rho, \quad (2)$$

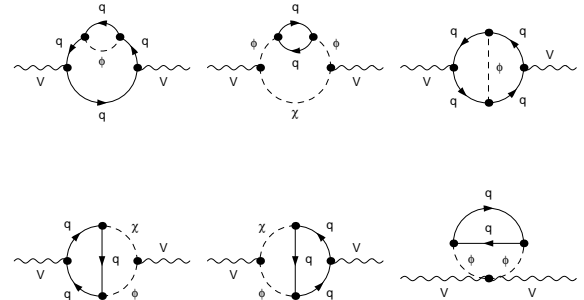
with  $c_W^2 = 1 - s_W^2 = M_Z^2/M_W^2$ . Within the MSSM the corrections have so far been restricted to  $O(\alpha\alpha_s)$ , see Ref. (3). In order to match the accuracy obtained in the SM and the (prospective) experimental uncertainties, the leading two-loop corrections to  $\Delta\rho$  at  $O(G_F^2 m_t^4)$  are desirable. These corrections are the lowest order contributions involving non-SM particles with a  $m_t^4$  dependence. In the limit of a large SUSY scale,  $M_{SUSY} \gg M_Z$ , the SUSY particles decouple from the observables, leaving the two

Higgs doublets of the MSSM active. First results for this contribution have recently been calculated in Refs. (4, 5).

Feynman-diagrammatic two-loop calculations involve a large number of diagrams. In the MSSM the additional problem of a proliferation of scales is apparent. Therefore, in order to perform the calculation as presented in Ref. (4, 5), the use of computer algebra programs is inevitable. In particular, we made use of the amplitude generator *FeynArts*, see Ref. (6), where the MSSM model file has recently been completed. The reduction of the amplitudes to scalar integrals have been performed with the program *TwoCalc*, see Ref. (7).

## TECHNIQUES FOR TWO-LOOP CALCULATIONS

In order to calculate the  $O(G_F^2 m_t^4)$  corrections to the  $\rho$ -parameter, the diagrams in Fig. 1 have to be evaluated.



**FIGURE 1.** Generic diagrams for  $O(G_F^2 m_t^4)$  corrections to  $\Delta\rho$ . ( $V = Z, W$ ,  $f = t, b$ ,  $\phi, \chi = h, H, A, H^\pm, G, G^\pm$ )

The diagrams and the corresponding amplitudes have been generated with the help of the *Mathematica* program *FeynArts* and the recently accomplished MSSM model file. This model file contains all relevant information

about the MSSM particles and vertices. (Only MSSM counter term vertices are missing at present.) *FeynArts* has been checked in several ways to insure its reliability. Apart from many self-energy calculations, whole processes like  $e^+e^- \rightarrow t\bar{t}$ ,  $e^+e^- \rightarrow H^+H^-$ ,  $q\bar{q} \rightarrow t\bar{t}$  and  $e^+e^- \rightarrow \tilde{\chi}^{0,+}\tilde{\chi}^{0,-}$  have been calculated by hand and with *FeynArts*. Perfect agreement has been found for all processes, see Refs. (6, 8).

How easy the evaluation of the amplitudes has become with *FeynArts* is demonstrated in the following sequence from a *Mathematica* session where the Z boson self-energy amplitude, corresponding to Fig. 1, has is obtained. (A detailed guide can be found in Ref. (6).)

---

```
(start of Mathematica session)
> <<FeynArts.m;
  (→ loading FeynArts into Mathematica)

> se2 = CreateTopologies[2, 1->1,
  ExcludeTopologies->Internal];
  (→ two-loop topologies are created)

> V2V2 = InsertFields[se2,
  V[2] -> V[2],
  Model->"MSSM",
  ExcludeParticles -> {...} ];
  (→ fields are inserted into the topologies, incoming
  field V[2] = Z and outgoing field V[2] are specified,
  the model is chosen to be the MSSM)

> Paint[V2V2];
  (→ the diagrams are painted (optional), see Fig. 1)

> V2V2A = CreateFeynAmp[V2V2];
  (→ Feynman diagrams are converted into amplitudes)

> V2V2A >> V2V2.amp;
  (→ result for the two-loop amplitude of the Z self-
  energy is saved in the file V2V2.amp)
(end of Mathematica session)
```

---

The further evaluation of the amplitudes has been performed with the *Mathematica* program *TwoCalc*, see Ref. (7). It performs the reduction of the self-energy amplitude at the two-loop level to a basic set of scalar integrals. The application of *TwoCalc* is demonstrated in the following sequence from a *Mathematica* session where the Z boson self-energy amplitude is processed.

---

```
(start of Mathematica session)
> <<TwoCalc.m;
  (→ loading TwoCalc into Mathematica)

> amp = <<V2V2.amp;
  (→ the amplitude obtained with FeynArts is loaded)

> SetOptions[ TwoLoop,
  CollectFunction -> 0];
```

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(→ options are set. *CollectFunction* allows to choose between DREG and DRED)

```
> res = TwoLoopSum[amp,
  SelfEnergyPart -> 1];
  (→ the amplitude is reduced to scalar integrals)

> res = Collect[res,
  {A0[_], T[_]}, Simplify];
  (→ the result is simplified (optional))

> res >> V2V2.res;
  (→ the result for the Z boson self-energy amplitude in
  terms of scalar integrals is saved in V2V2.res)
(end of Mathematica session)
```

---

The analytically obtained result now consists of scalar integrals at the one- and at the two-loop level. In the case presented here, since the external momentum is set to zero, these are the functions  $A_0$  and  $T_{134}$  at the one- and two-loop level, respectively, see Refs. (9, 10).

The pure two-loop diagrams have to be supplemented with the corresponding one-loop diagrams with counter term insertion. In order to obtain the  $O(G_F^2 m_t^4)$  corrections an expansion of the amplitudes up to  $O(m_t^4/M_W^4)$  had to be performed. All remaining  $M_W = M_{ZC_W}$  have been set to zero. Furthermore we had to apply the MSSM sum rules for Higgs boson masses, especially implying for the case  $M_Z, M_W \rightarrow 0$  that the lightest MSSM Higgs boson has the mass  $m_h = 0$  at tree level. After adding up all contribution, the expansion of the result in terms of  $\delta = (4 - D)/2$  leads to a finite result in the limit  $\delta \rightarrow 0$ . All relevant details can be found in Refs. (4, 5).

## RESULTS FOR $\Delta\rho$ AND $M_W$

After extracting the prefactor of  $m_t^4/M_W^4$  and setting  $M_W$  to zero, besides  $s_\beta \equiv \tan\beta/\sqrt{1+\tan^2\beta}$  only two mass scales remain: the top quark mass,  $m_t$ , and the mass of the  $C\mathcal{P}$ -odd Higgs boson,  $M_A$ . The result for  $\Delta\rho$  can be conveniently expressed in terms of  $a \equiv m_t^2/M_A^2$ :

$$\Delta\rho_1^{\text{SUSY}} = 3 \frac{G_F^2}{128\pi^4} m_t^4 \frac{1-s_\beta^2}{s_\beta^2 a^2} \times$$

$$\left\{ \text{Li}_2\left(\left(1-\sqrt{1-4a}\right)/2\right) \frac{8}{\sqrt{1-4a}} \Lambda \right.$$

$$- 2\text{Li}_2\left(1-\frac{1}{a}\right) [5-14a+6a^2]$$

$$+ \log^2(a) \left[1 + \frac{2}{\sqrt{1-4a}} \Lambda\right] - \log(a) [2-20a]$$

$$\left. - \log^2\left(\frac{1-\sqrt{1-4a}}{2}\right) \frac{4}{\sqrt{1-4a}} \Lambda \right\}$$

$$\begin{aligned}
& + \log \left( \frac{1 - \sqrt{1 - 4a}}{1 + \sqrt{1 - 4a}} \right) \sqrt{1 - 4a} (1 - 2a) \\
& - \log(|1/a - 1|) (a - 1)^2 \\
& + \pi^2 \left[ \frac{2\sqrt{1 - 4a}}{-3 + 12a} \Lambda + \frac{1}{3} - 2a^2 \frac{s_\beta^2}{1 - s_\beta^2} \right] \\
& - 17a + 19 \frac{a^2}{1 - s_\beta^2} \Bigg\} \quad (3)
\end{aligned}$$

with  $\Lambda = 3 - 13a + 11a^2$ . As a consistency check, in the limit of  $M_A \rightarrow \infty$ ,  $a \rightarrow 0$ , we obtain

$$\Delta\rho_1^{\text{SUSY}} = 3 \frac{G_F^2}{128\pi^4} m_t^4 [19 - 2\pi^2]. \quad (4)$$

This is the SM result with  $M_H \rightarrow 0$ . It shows that the MSSM decouples to the SM limit (also at the two-loop level) when the new scales, here  $M_A$ , is made large.

In the limit of large  $\tan\beta$  (i.e.  $(1 - s_\beta^2) \ll 1$ ) one obtains

$$\Delta\rho_1^{\text{SUSY}} = 3 \frac{G_F^2}{128\pi^4} m_t^4 \left[ \frac{19}{s_\beta^2} - 2\pi^2 + o(1 - s_\beta^2) \right]. \quad (5)$$

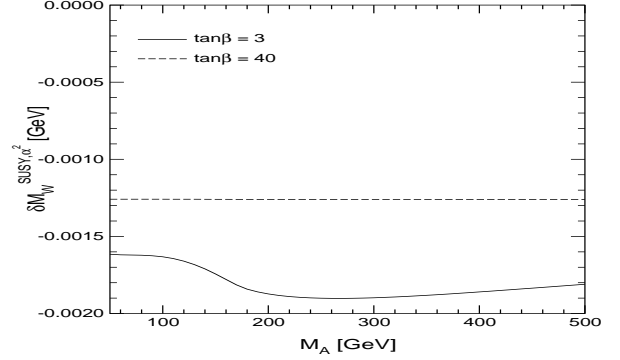
Thus for large  $\tan\beta$  the SM limit with  $M_H \rightarrow 0$  is reached.

The numerical result obtained for the  $O(G_F^2 m_t^4)$  corrections to  $\Delta\rho$  vary around  $-2 \times 10^{-5}$  and are of the same size (but with opposite sign) as the leading  $O(\alpha\alpha_s)$  MSSM corrections originating from the scalar top and bottom sector, see Ref. (3), if the common soft SUSY-breaking scale is chosen to be  $M_{\text{SUSY}} \approx 500$  GeV. For a larger value,  $M_{\text{SUSY}} \approx 1000$  GeV, the new  $O(G_F^2 m_t^4)$  corrections are about three times larger than the  $O(\alpha\alpha_s)$  contributions. Furthermore it is well known that the  $O(G_F^2 m_t^4)$  SM result with  $M_H^{\text{SM}} = 0$  underestimates the result with  $M_H^{\text{SM}} \neq 0$  by one order of magnitude. One can expect (see Ref. (4)) a similar effect in the MSSM once higher order corrections to the Higgs boson sector are properly taken into account, which can enhance  $m_h$  up to  $m_h \lesssim 130$  GeV, see Ref. (11).

With the help of eq. (2) the shift in the  $W$  boson mass can be evaluated. In Fig. 2 the induced shift for  $M_W$  is shown as a function of  $M_A$  for  $\tan\beta = 3, 40$ . The effect on  $\delta M_W$  varies between  $-1.5$  MeV and  $-2$  MeV for small  $\tan\beta$  and is almost constant,  $\delta M_W = -1.25$  MeV, for  $\tan\beta = 40$ . The constant behavior can be explained by the analytical decoupling of  $\tan\beta$  in eq. (3) when  $\tan\beta \gg 1$ , see eq. (5).

## CONCLUSIONS

We presented the calculation of the SUSY contributions of  $O(G_F^2 m_t^4)$  to the  $\rho$ -parameter. In order to obtain these two-loop corrections, the computer algebra programs *FeynArts* and *TwoCalc* have been used. Examples



**FIGURE 2.** The induced shift in  $M_W$  is shown as a function of  $M_A$  for  $\tan\beta = 3, 40$ .

of the handling of these programs were given. A compact analytical result in terms of  $m_t^2/M_A^2$  has been derived. The induced shift in  $M_W$  varies between  $-1$  MeV and  $-2$  MeV, depending on  $M_A$  and  $\tan\beta$ .

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